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Geometric Thinking Behaviours of Undergraduates on-Entry and at-Exit of Online Geometry Course

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Abstract: The development of students' geometry thinking through digital platforms remains quite debatable in pedagogical literature. This descriptive study examined the entry and exit geometric thinking behaviours of students who took an undergraduate Geometry course via *vclass* Moodle platform for the first time. The 14-week course was designed and delivered using Gagne's nine events of instructions. Data were collected from a cohort of 280 first year students pursuing degree of Bachelor of Science in Mathematics Education at the University of Education, Winneba. The van Hiele's Geometric thinking test was used to assess entry and exit behaviours of participants. Data were first analysed descriptively and displayed in proportions and charts. Paired samples t-test was carried out to test for significant differences between entry and exit geometric thinking behaviours of participants. On-entry, the result shows that close to three-quarters of the students operated at the lowest level of geometric thinking i.e. visualization. Only about 20% and 6% exhibited analysis and abstraction skills respectively while no participant demonstrated the highest thinking skills of geometric deduction and rigor. At-exit, the proportions of students increased markedly and differed substantially from entry across geometric thinking levels. The difference in students' geometric thinking behaviours between entry and exit were statistically significant. The study concludes that delivering undergraduate geometry course via the *vclass* Moodle platform significantly improves undergraduates' geometric thinking skills despite some observed constraints. It is recommended that lecturers design their undergraduate geometry course on *vclass* in line with Gagne's nine events of instructions to bridge existing thinking gaps.

Keywords: van Hiele's, Geometric Thinking, UEW Online Moodle: Vclass

1. Introduction

The importance of Geometry thinking cannot be overemphasized in undergraduate mathematics education. By studying geometry, students are able to progressively develop critical thinking skills, spatial abilities and problem solving skills. Geometry thinking is also necessary for the understanding of algebra and calculus [1]. To develop students' progressive understanding, the geometry content is often designed to ensure spiral learning trajectory from basic to advanced geometric thinking levels [2, 3]. In Ghana for example, the Geometry curriculum is designed such that students entering undergraduate geometry course would have the prerequisite skills of visualizing basic shapes, analyzing and abstracting figural properties and making formal deduction of geometric entities with rigor. With this, students undertaking undergraduate Geometry could transition smoothly to higher intuitions, formal definitions, axioms, postulations and proofs in geometry and apply such skills in other mathematics courses [3].

Geometric thinking of students has been theorized by different authors. Some of these are Piaget and Inhelder's topology of geometric thought, Duval' theory of figural apprehension, Fischbien's theory of figural concept, the dimensional deconstruction theory and the theory of spaces for geometric work and the van Hieles' theory of geometric thinking [4]. Van Hiele's theory for example categorized students' geometric thinking into five hierarchical thinking levels [4-7]. The 1st level, visualization, is where students think about basic geometric concepts in shapes holistically, through visual considerations and describe geometric figures by only the visual attributes and irrelevant properties. The 2nd level is analysis, where students reason about geometric concepts by means of informal observation of attributes to determine or establish necessary properties of shapes. The 3rd level is the abstraction where students logically order the properties of concepts, construct abstract definitions and distinguish between necessary and sufficient properties of a concept. The 4th level is deduction. At this level, students think in a formal way within the context of a mathematical system, undefined theorems, axioms, an underlying logical system, definitions, and theorems. The 5th and final level is the rigor. At this level, students study various geometries by comparing axiomatic systems in a more formal, theoretical way usually without concrete models. By this van Hiele's categorization, learners tend to develop gaps if their prerequisite thinking level is inadequate. Studies argue that most learning difficulties experienced by students emanate from gaps in their van Hiele's geometric thinking levels [8]. In this study, the development of undergraduate students' geometric thinking behaviours were examined through the lens of van Hiele's geometric thinking levels. Therefore, the aim of this study was to investigate the geometric thinking behaviour of first year mathematics students who took first course in Geometry via *vclass* and whether there are any differences in their on-entry and at-exit geometric thinking behaviour.

2. Literature Review

Research has shown that the development of students' geometric thinking is largely dependent on the teaching and learning environment employed [9, 10]. Globally, there is a growing worry about how teachers teach Geometry [2], how students learn the subject [1] and what learning environment facilitates students' geometric thinking [11, 12]. Akayuure, et al [1] reported that the use of origami in face to face instructional environment is effective in developing students' spatial thinking in geometry. The use of dynamic software such as Geogebra has also proven to be effective in developing students understanding of geometry. Despite these evidences, research remains inconclusive about whether the recent emerging digital platforms such as *vclass* promotes geometry thinking skills. This study therefore sought to examine how the use of *vclass* improves undergraduates' geometric thinking skills.

Over the years, mathematics and geometry instruction has always thrived in the conventional face to face mode. However, in recent times some mathematics researchers and educators have advocated for the teaching and learning of mathematics via technology-rich virtual platforms such as Moodle, Sakai, Zoom, Edmodo, Quipper, Google meet, WebCT, Desire@Learn, Google Hangout, Google Classroom, Blackboard, Olat and WebEx [13]. The advocacy has been grounded on new learning preferences of digital-age generation Z learners and some research findings that suggest that virtual platforms promote better mathematics achievement and ensure the development of life-long learning skills [14]. Zulu, et al [13] for example, reported that ZOOM app facilitates group work, allows lecturers to schedule mathematics lessons in advance and enables students to learn favourably from anywhere, anytime at their own pace.

Other researchers have however enumerated several challenges associated with the use of virtual platforms for teaching and learning mathematics [13, 15]. These challenges ranged from network connectivity, non-availability of mathematical symbols, limited capacity to accommodate large class size during online delivery, power outages, lack of technological knowledge, data issues, to lack of devices for online learning [13]. In such varied viewpoints, it remains a contention when comparing virtual platforms to the orthodox face to face environments. This contention leaves a void in our understanding of how the use of *virtual platforms* could improve students' mathematical or geometric thinking which relates more to the *physical world* than *virtual space*.

For the past decades when most countries enacted their ICT in Education policies to promote the integration of 21st century technologies in teaching, teachers have been cautious in deploying emerging virtual platforms for mathematics instruction. However, in March 2020 when the novel coronavirus (COVID-19) pandemic hit the world, triggered lockdowns and disrupted the face to face instructions, many institutions of higher learning were compelled to move lectures to virtual platforms [16]. In April 2020 for example, mathematics instructional delivery at the University of Education, Winneba (UEW) in Ghana, moved to a new Moodle based virtual learning management system (LMS) dubbed UEW *vclass*. Similar to many other educational institutions across the world, the move was taken without recourse to any previous evidence of how effective the *vclass* platform supports academic work. The rushed into the *vclass* caused both mathematics lecturers and students to struggle to bring the 2019/2020 academic year to a close. In fact, as first time users of *vclass*, Akayuure [17] reported that lecturers had challenges designing mathematical tasks, delivering content virtually and assessing students online. Students also experienced problems navigating the platform, retrieving content and completing assessment online. Within these challenges, it seems unclear if the use of the *vclass* is promoting the required mathematical thinking among students. Specifically, for this study, the key question is whether geometry lessons delivered on *vclass* improve students' geometric thinking.

Previous literature has documented the worth of *vclass* platform in learning. An analysis by Akayuure [17] revealed that the use of *vclass* in Moodle has the potential of promoting remote learning. There is however, little information about how the use of *vclass* affects students' mathematical thinking and particularly, whether first time users of *vclass* experience any positive changes in their geometric thinking behaviours. This present study examined the geometry thinking behaviours of first year mathematics students who took first undergraduate Geometry course via *vclass*. Specifically, the study examined changes in the geometric thinking behaviour of first year mathematics students who took first course in Geometry via *vclass* and whether there are any differences in the on-entry and at-exit geometric thinking behaviour students' in the first course in Geometry.

The significance of the study hinges on our conviction that learning in digital space has become imperative in higher institutions at the wake of the emerging digital technologies and protocols on limited human physical contacts resulting from the recent COVID-19 pandemics. At the heart of this study is how geometry lecturers can design and utilize digital environments to promote geometric thinking among generation Z undergraduates whose daily lives revolve around digital materials of the fourth industrial revolution (4IR). The findings would therefore provide educators, policymakers and others stakeholders with empirical evidence regarding how the use of digital platforms such as *vclass* affects students' geometric thinking levels. Findings of the study would also give a picture of the effect of geometric thinking trajectory of students transitioning into and exiting undergraduate geometry. This would help in decision making regarding instructional designs and practices, and support systems needed to improve the learning of undergraduate geometry.

2.1. Geometric Thinking and Technology Integration

The teaching of geometry in the fourth industrial revolution has shifted from the use of a conventional approach to the use of dynamic software and recently digital technology platforms. Teaching and learning of geometry with technology have been widely recognised. Abidin [18] reviewed the development of students' geometric thinking based on technology, and listed a number of suitable technologies such as GeoGebra, Smartboard, Game-based interactive Whiteboard, Geometer Sketchpad, Touch screen, Virtual math team, Google Sketch Up and TI-Nspire. Adulyasas and Abdul Rahman [19] conducted a study on a lesson study incorporating phase-based instruction using Geometer's Sketchpad (LS-PBI) and its effects on Thai students' geometric thinking. Their result showed that the LS-PBI was effective in improving students' geometric thinking. Govender and Govender [20] conducted a study on students' knowledge of technologies when teaching circle geometry through a careful implementation of geometric habits of mind instructional design mode. They concluded that all of the students liked the readily available online learning tool. Another study on the geometric thinking using educational video gaming found that the software supported the geometry learning among blind students, though this remains complicated issue [21].

Kurniawan, et al [22] developed android-based comic learning media to describe the increase in geometric thinking skills of students who used it. The result revealed that students' level of thinking had increased. Adulyasas and Yathikul [23] also developed the tablet application for enhancing geometric thinking of secondary students in learning geometry with van Hiele's phase based learning. They found that students' levels of geometric thinking was greater than learning without the tablet and the students were very satisfied with learning geometry on the application. Despite these, Naidoo [15] reported that many mathematics teachers are still hesitant in using technology in their classrooms even though there are many digital applications and websites that can support learning and improve mathematics achievement.

The literature reviewed above suggests that substantial work has been done on the use of different geometric software in developing geometric skills of students. However, in terms of virtual learning management systems such as *vclass* in Moodle, literature is very scanty. The present study therefore fills this void by examining how learning undergraduate geometry via *vclass* in Moodle platform could improve the geometric thinking behaviour of students.

2.2. Instruction Design

The design of courses on learning management systems usually follow an underlying instructional design model or learning theory. Some of the models include Merrill's First Principle of Instruction, Gagne nine events of instruction, and Analysis, Design, Development, Implementation and Evaluation (ADDIE) model. In this study, the Gagne's nine events of instructions would be applied in the design of geometry lessons in *vclass* in Moodle platform. This was because literature describes Gagné's approach as an ideal one to use for the incremental lesson delivery in which students learn about and apply their knowledge. The Gagne's nine events of instructions has been a widely applied instructional design in virtual platform because it allows both the instructor and the student to systematically follow the learning trajectory within the virtual environment [24]. The nine events are (1) to gain students' attention, (2) to inform students of the objectives for each lesson, (3) to stimulate recall of prior learning, (4) to present the content, (5) provide guidance, (6) to elicit performance, (7) to provide feedback, (8) to assess learning and (9) to transfer learning to new context [25].

3. Methodology

3.1. Design

The study was a descriptive survey utilizing action research design to investigate the geometric thinking levels of undergraduate preservice teachers before and after undertaking geometry course. The geometry course covered formal notions and proofs of properties of plane shapes such as rectangle, square, quadrilaterals and polygons. As a prerequisite to the course, participating students had sound knowledge of basic definitions and properties of common plane shapes and space.

3.2. Participants

First year students for 2019/2020 academic year at the Department of Mathematics Education, University of Education, Winneba, were purposively selected for the study. The choice of this category was to allow for detection of transition knowledge level as well as effect of *vclass* instruction on students' geometric thinking. The sample comprised 280 students from six groups of first year cohort offering first undergraduate Geometry course.

The participants comprised 81.1% males and 18.2% females with .7% not indicating their gender identity. In terms of age, majority of the participants (72.1%) were within the early adulthood ages of 20 and 29. About 11% were below 20 years and 17% were above 30 years. There were also approximately 45% who entered the university with diploma certificates in teaching and close to 55% who entered the university using senior high school (SHS) certificates. This result indicates that the participants have heterogeneous background characteristics with some having experiences in teaching at the basic school level in Ghana.

3.3. The Undergraduate Geometry One Course on Vclass

The undergraduate course was designed to develop students' knowledge for teaching basic definitions and proofs of geometric figures, properties of triangles, circle theorems, secants, tangents and normal to a circle, segment ratio, algebraic description, geometric construction, properties, nets, areas and volumes of solid shapes. The course was 14-weekly lecture sessions designed on *vclass* of the University of Education Winneba Learning Management System (UEW-LMS).

For the study, the design principle and pedagogical approach on the *vclass* followed Robert Gagne's nine events of instructions [26]. The first was to *gain students' attention* through a humorous video clip, a controversial discussion or a challenge. The second was to *inform* students of the objectives for each lesson. The third was to provide students with opportunity to *stimulate recall* of prior learning through short quizzes, chat rooms and wikis. The fourth was to *present the content* of the topics to students through PowerPoint lecture notes, video recordings and real-time interactions/discussions.

The fifth was to provide *guidance* and precise instructions and descriptions of hyperlinks, clickable and navigation buttons to enable students follow what next to do, download study materials. The sixth was to *elicit* performance by providing time for practice through eLearning branching scenarios, quizzes, assignments, group sharing and projects. The seventh entailed the provision of timely *feedback* through peer-evaluation, scoring rubrics, posting of short messages to students' portals, emails and group WhatsApps pages. The eighth was to *assess* performance. This was done through pop up notice about time and duration for quizzes and mid-semester and end of semester examinations. The ninth and final step entailed enhancing retention and *transfer of learning*. This was done by embedding real world scenarios, YouTube videos and linkages with undergraduate *Geometry Two course* which were to be taken in the ensuing semester.

3.4. Instrument

The van Hiele geometric Test (VHGT) was adopted and used to collect data on both entry and exit geometric thinking behaviour of participants. The VHGT was developed and copyrighted by Zalman Usiskin and Susan Senk for the Cognitive Development and Achievement in Secondary School Geometry project in 1982. The test, which is now open released, contains 25-item multiple-choice test items ordered sequentially in blocks of five van Hiele levels comprising five questions for each level. Items 1 to 5 measures visualization skills, item 6 to 10 analysis skills, item 11 to 15 abstraction skills, items 16 to 20 deduction skills and item 21 to 25 rigor in van Hiele level. The VHGT has been accepted and widely used in literature without questionable validity and reliability [5, 7]. Hence, its adoption in this study.

3.5. Data Collection Procedure

The data collection was done in two phases. The Geometry Lecturer was informed of the purpose of the study through a verbal discussion and a letter of introduction from the Head of Department of Mathematics Education, University of Education, Winneba. This was followed by visits and interaction with students in their respective lecture halls on the first day of their Geometry lecture to seek for consent to participate in the study. The students were informed of the need to answer the questions by themselves without seeking for assistance or giving assistance to a colleague. They were also assured of their confidentially and that fact that the test does not contribute to their grading in the course. The VHGT was administered during the first class of the semester in each of the six groups.

The first day of lectures was considered appropriate to ensure that we were measuring the geometric knowledge participants entered the university with. Participants were asked to follow instructions provided in the test paper. The Geometry lecturer assisted in supervising and collecting the scripts. Participants were allowed ample time to complete the test items and were allowed to submit their scripts when they were satisfied of their responses. The second administration of the test was done at the last day of lectures in the course which lasted for 14 weeks. Similar approach was adopted to administer VGHT under the supervision of the researcher and the Geometry course lecturer.

3.6. Data Analysis

Data on participants were coded and keyed into SPSS for further processing and analysis. Data were then analysed descriptively using frequency count and percentages displayed in tables and bar chart. The paired sampled t-test was used to test any significant difference between participants' entry and exit geometric thinking levels. The test were done at .05 significance level. Assumptions of independence of subjects, paired measurement of variable and particularly test for normality were checked and data showed no serious violation for the assumptions needed to apply paired sample t-test.

4. Results

4.1. Comparison of Entry and Exit Thinking Behaviours of Participants

The first objective of the study was to examine the changes in preservice mathematics teachers' geometric thinking behaviour on-entry and at-exit of undergraduate geometry course taught in *vclass*. The geometric thinking behaviour was operationalized as participants' skills of visualizing, analysing, abstracting, deducting and rigor as described in van Hiele's theory.

4.1.1. Visualization Skills

Visualization skills in this study refers to the ability to recognize figural orientations, discriminate and describe squares, triangles, rectangles or parallelograms holistically by only their visual attributes. The distribution of proportions of participants' correct application of their visualization skills in 5 items of VHGT test on-entry and at exit of geometry course is shown in Table 1.

Table 1. *Percentage distribution of participants' visualization skills.*

Level	Item #	Aspect of Geometry tested	On-entry	At-exit	%Δ
			Correct	Correct	
VHL 1 Visualization	1	Discrimination of squares	89.5%	90.1%	0.6%
	2	Discrimination of triangles	94.3%	100.0%	5.7%
	3	Discrimination of rectangles	86.7%	98.2%	11.5%
	4	Description of orientation of squares	46.6%	60.1%	13.5%
	5	Recognition of orientation and class inclusivity of parallelogram	37.5%	49.3%	11.8%

As shown in Table 1, *on-entry*, more than 86% of participants could visually recognize and discriminate correctly between squares, triangles and rectangles. The proportions of participants increased to above 90% at-exit of the geometry course which was taught via *vclass*. On the contrary, less than 50% of the participants were able to recognize the correct orientation of different squares and parallelogram using class inclusivity property. *At-exit*, these proportions increased substantially by 13.5% and 11.8% respectively. This means majority of the participants were able to visualize geometric figures prior to the start of the course and have consolidated their abilities after the course.

4.1.2. Analysis Skills

Geometric analysis skill involves ones' ability to reason and use informal observations to determine and establish essential properties of shapes such as square, rectangle, rhombus, triangle and kite. Table 2 shows percentage distribution of participants who correctly used their analysis skills to respond correctly to VHGT test.

Table 2. *Percentage distribution of participants' analysis skills.*

Level	Item #	Aspect of Geometry tested	On-entry	At-exit	%Δ
			correct	correct	
VHL 2 Analysis	6	Relationship property of square	48.6%	85.9%	37.3%
	7	Diagonal property of rectangle	21.3%	78.1%	56.8%
	8	Properties of rhombus	37.9%	42.2%	4.3%
	9	Properties of isosceles triangle	86.4%	97.9%	11.5%
	10	Properties of kite	37.8%	60.1%	22.3%

As shown in Table 2, *on-entry* to the course, less than 50% could analyse correctly the relationship property of square, diagonal property of rectangles and basic properties of rhombus and kite. However, up to 86% of participants could establish correctly the basic properties of an isosceles triangle. *At-exit*, the proportions of participants who correctly analysed the properties of square, rectangle, rhombus, triangle and kite increased markedly by 37.3%, 56.8%, 4.3%, 11.5% and 22.3% respectively. The result suggests that majority of the participants could analyse geometric properties and establish relationships among shapes after the course.

4.1.3. Abstraction Skills

Abstraction or ordering skills involve the ability to logically order, construct abstract definitions and distinguish between necessary and sufficient properties of a shape. Table 3 displays percentage distribution of participants who abstracted correctly in the VHGT test.

Table 3. *Percentage distribution of participants' abstraction skills.*

Level	Item #	Aspect of Geometry tested	On-entry	At-exit	%Δ
			correct	correct	
VHL 3 Abstraction	11	Verbal logics reasoning with rectangle/triangle	24.1%	40.7%	16.6%
	12	Logical reasoning with triangle property	51.4%	56.7%	5.3%
	13	Abstracting with orientation of rectangle	37.7%	53.0%	15.3%
	14	Logical argument with inclusive properties	10.1%	57.0%	46.9%
	15	Logical relationships between parallelograms	22.9%	49.9%	27.0%

From Table 3, the proportions of participants (24.1%, 51.4%, 37.7% and 22.9%) who correctly demonstrated verbal logical reasoning and logic on shape and space were very low *on-entry* to the course. *At-exit*, the proportions of participants who responded correctly to all items on geometric ordering remained very low but only increased by about 17%, 5%, 15% 47% and 27% respectively. Despite some substantial changes in reasoning verbally and logically, the proportions of participants who abstracted correctly still remained low. This means majority did not substantially improve upon their logical and verbal reasoning after the course.

4.1.4. Deduction Skills

Deduction skills entail formal reasoning in the context of a geometric system, undefined theorem, axiom, an underlying logical system, definition and theorem. Table 4 is the distribution of percentage of participants who were able to reason deductive to answer the VHGT test items correctly.

Table 4. *Percentage distribution of participants' deduction skills in shapes.*

Levels	Item #	Aspect of Geometry tested	On-entry correct	At-exit correct	%Δ
VHL 4 Deduction	16	Deduction of embedded figural construction	19.8%	50.1%	30.3%
	17	Deduction of figural properties	17.5%	38.8%	21.3%
	18	Proof	20.4%	55.2%	34.8%
	19	Generalization	10.9%	35.9%	25.0%
	20	Deduction	23.2%	36.2%	13.0%

From Table 4, less than a quarter of participants (19.8%, 17.5%, 20.4%, 10.9% and 23.2%) responded correctly to the five items on geometric deduction. However, *at-exit*, the proportions (30%, 22%, 35%, 20% and 13% respectively) responding correctly to the five items on deduction of figural properties were comparatively high. Despite the increases, it can be seen that the proportions of participants remained appreciably lower.

4.1.5. Rigor in Geometry

In this study, rigor in geometry thinking refers to the ability to make comparison of axiomatic systems in a more formal or theoretical way without concrete models. Participants' responses to items 21 to 25 were analysed to determine the proportions of participants who could think with rigor when dealing with geometry conclusions about a theory, an axiom or an implicative statement. The results is presented in Table 5.

Table 5. *Percentage distribution of participants' rigor in geometry thinking in shapes.*

Level	Item #	Aspect of Geometry tested	On-entry correct	At-exit correct	%Δ
VHL 5 Rigor	21	Deduction with rigor	29.8%	25.2%	-4.6%
	22	Deductive conclusion with rigor	18.9%	25.4%	6.5%
	23	Deduction with rigor	30.7%	33.3%	2.6%
	24	Deduction with rigor	38.7%	38.0%	-0.7%
	25	Geometric implicative	31.7%	34.8%	3.1%

As shown in Table 5, *on-entry* to the course, the proportions of participants who responded correctly to 21 to 25 were very lower but these increased *at-exit* only slightly across three items. Surprisingly, the proportions of two of the items *at-exit* were rather less than *on-exit*. This means participants still have challenges with rigor in geometric construction and generalization of implicative statements.

Figure 1 shows the overall gap in geometric thinking behaviour of students between the on-entry and at-exit of the course.

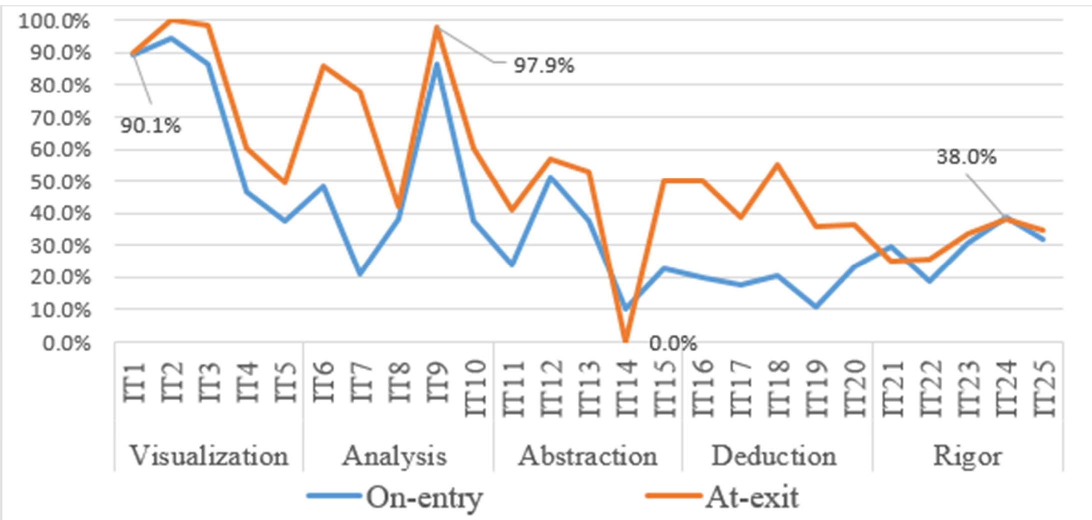

Figure 1. *Participants' geometric thinking on-entry and at-exit of geometry course.*

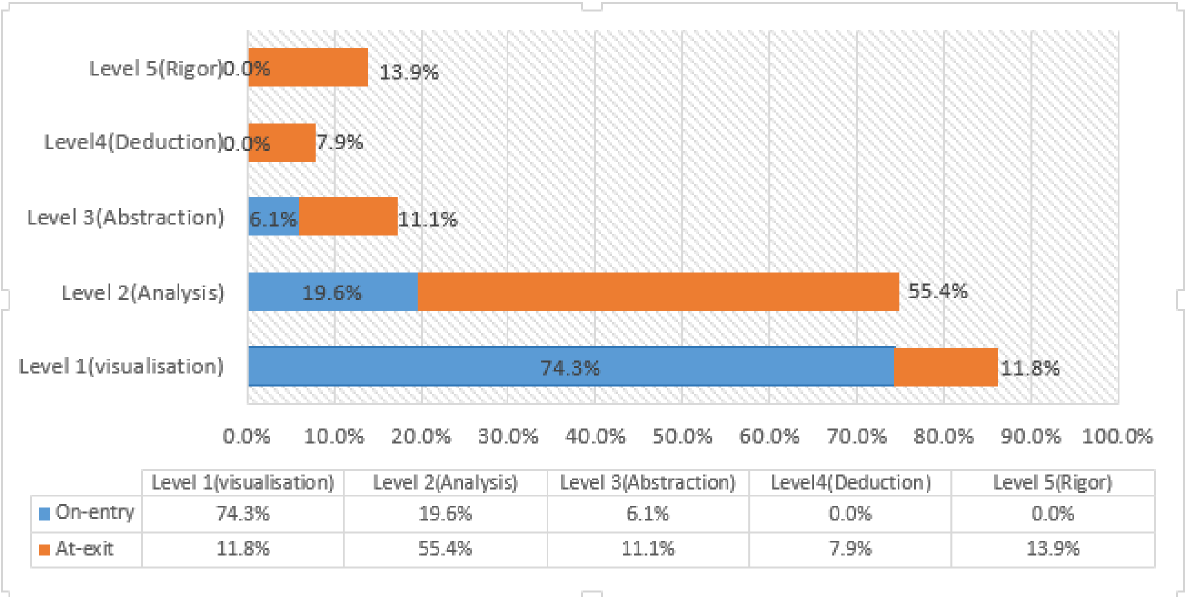

Figure 2. *Entry and exit geometric thinking levels of participants.*

From Figure 1, the result shows a decreasing trend of the thinking behaviour of the students as one move from the lower level of visualization to rigor. It also reveals that except for abstraction IT14, more students gained in their thinking behaviour after the course. Particularly, the proportion of students who were able to reason deductively about theorems in geometry increased substantially from 20.4% to 55.2%.

4.2. Difference in Overall Thinking Behaviour on-Entry and at-Exit of the Course

Figure 2 shows the comparison of proportions of students who attained each level of geometric thinking based on van Hiele's criteria of categorization.

On-entry to the undergraduate Geometry course, the result in Figure 2 shows that about 74% of participants exhibited visualization skills and nearly 20% and 6% operated at analysis and abstraction levels respectively. Unexpectedly, none of the participants reached deduction and rigor levels on-entry for the geometry course.

At-exit of the course, the result shows relative progression in geometric thinking across levels. Only 11.8% remained at visualization level while those who attained analysis level increased on-entry from 19.6% to 55.4% at-exit. Similarly, the proportions of participants who reached abstraction level on-entry increased by about 4% at-exit. Finally, while no participant attained deduction and rigor levels on-entry to the course, about 8% and 14% attained deduction and rigor levels respectively.

Further analysis was done using paired sample t-test to test any statistical significant difference between participants' geometric thinking on-entry and at-exit of the course. The result is displayed in Table 6.

Table 6. *Paired samples test of participants' scores in entry and exit tests.*

Tests	Paired Differences				t	df	p
	Mean	SD	95% CI of Dif.				
			Lower	Upper			
Exit test – Entry test	6.5	1.09	5.24	7.83	9.937	274	.000

The result of the paired sample t-test in Table 6 indicates the mean difference of 6.5 with the lower and upper bounds of 5.24 and 7.83 at 95% confidence interval of the difference. The analysis yielded $t(274) = 9.937$ corresponding to $p < .001$. This shows that at .05 significance level, there is statistically significant difference in the test scores between on-entry and at-exit of the geometry course. Participants therefore did significantly better at-exit compared to on-entry of the course.

5. Discussion

The purpose of this study was to examine the entry and exit geometric thinking behaviour of the mathematics education students who learnt undergraduate geometry via *vclass*. Framed within the van Hiele's geometric thinking levels, this study focused on the students' progression from visualization, analysis, abstraction, and deduction towards geometric rigor.

The study found that *on-entry* to the undergraduate geometry course at the university, majority of participants were operating within visualization level with few reasoning at the analysis and abstraction levels, and none at deduction or rigor levels. This means that participants could only discriminate different orientations of rectangles, rhombuses and kites using the class inclusivity properties. This suggests that participants' entry geometric thinking behaviour was lower than the abstraction level recommended for entering into undergraduate course in Ghana [3, 8, 11]. *At-exit* of geometry course which was delivered via *vclass*, it was found that up to 55% transitioned to the analysis level with few also reasoning at abstraction, deduction and rigor levels. This finding suggests that the course improved participants' thinking levels. However, cumulatively only 39.9% of the students are demonstrating sound visualization and analysis skills after the course. The exit geometric thinking behaviours therefore suggest that some students still have problems in abstracting, deducing and operating with rigor in geometry.

Even though, majority in the study could visualize, more than 50% still have problems with visualizing figural orientation and class inclusivity of plane shapes. Among the plane shapes, majority have problem with properties of rhombus and kites both on-entry and at-exit of the Geometry course. This problem could be traced to how students are usually introduced to shape and space at the basic school level or how the lecturer exposed participants to the geometry course via *vclass*. In geometry education, it is expected that teachers would teach shape and space logically from enactive through iconic to symbolism or from real to abstract [1]. However, it appears the *vclass* did not create sufficient interactive visualization effect for students leading to deficits in reasoning and construction of the geometric entities. The use of *vclass* in this study also did not seem to have provided adequate opportunity for students to experience, manipulate and discriminate shapes in different forms, sizes and orientations. This tends to affect participants' ability to visualize, describe, analyse and make deduction about shapes and their properties.

From the study, participants' geometric abstraction remains problematic. Majority of the participants failed to engage in correct verbal logical reasoning, make deduction of embedded figural constructions or present correct proofs and generalization to given theorems. This corroborates previous findings in Ghana [8, 11] and further raises the question of how geometry is learnt or taught in Ghanaian schools. In the studies mentioned above, both preservice teachers and senior high school students were found to be operating at lower thinking levels than the expectations of the Ghanaian mathematics curriculum. It is worth highlighting that deductive and verbal reasoning form the basis for understanding and stating definitions, properties, axioms, postulates and other geometric objects in proofs in geometry. Therefore, students' lack of this verbal reasoning suggests that they are likely to struggle in explaining geometric objects and in studying higher geometry or applying geometric knowledge in such areas as trigonometry and vector algebra in their undergraduate programme.

The study also found that participants' geometric thinking were significantly better at-exit than on-entry of the course. This suggests that teaching via *vclass* had significant effect on geometric thinking of participants pursuing mathematics education programme. Similar findings have been reported in literature [1, 11] where considerable progress in participants' van Hiele geometric thinking levels were found after a designed geometry teaching lessons. However, as the significant effect is attributable to the teaching via *vclass*, it must be observed that there were challenges, such as lack of virtual manipulative and poor access to the *vclass* due to login and network problems encountered by both the lecturer and students during the online lesson deliveries. This could have affected the lecturer-students online interactions and consequently influenced the structure of the observed learning outcomes of participants.

6. Conclusion and Recommendation

The study concludes that most mathematics education students in this study did not seem to be ready for the first undergraduate geometry course. Furthermore, though at-exit, some participants still did not reach the higher level of deductive reasoning and rigor, the delivery of geometry course online via *vclass* improved the geometric thinking behaviour of undergraduates. However, with the current level of geometric reasoning, some of the students in this study are likely to struggle in learning further Geometry related courses.

Based on the findings of this study, the following recommendations are made:

1) Geometry lecturers should engage students in visual and verbal analysis of geometric shapes using precise definitions to ensure their readiness for and transition to undergraduate geometry course.
2) Senior high school curriculum should be designed to reflect van Hiele geometric thinking levels to ensure students' progress in their geometric thinking levels.
3) Lecturers of first undergraduate geometry course should first test and address the readiness of undergraduates for geometry course. Virtual manipulative and online models, multiple representations and dynamic constructions should be used to enable students to transition from lower to higher levels of geometric thinking. Lecturers also need to first revisit and revise senior high school geometry contents with their undergraduate students to bridge any gaps in their geometric thinking before proceeding to teach undergraduate course contents.
4) Lecturers should apply the van Hiele's theoretical framework and the Gagne's nine events of instructions in their design and delivery of geometry course via *vclass*.

7. Limitation

The study concedes two main limitations. First, the use of test alone to examine thinking behaviors appears inadequate. Perhaps, using interview data to triangulate the test scores could have provided richer information about participants' geometric thinking. Future study should therefore consider using mixed methods involving test and interviews to compare participants' written and oral thinking behaviors in geometry. Second, the design and use of the online mode of instruction for the first time might have affected the lecturer and students' interaction and output. Future study should consider a longitudinal study with small number of participants to allow for effective lecturer-students interaction through chats, forum discussions and real-time online communication.

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